order scheme. However, in VARPAR the second-order expressions are updated every period when predicting several periods ahead. Comparing Table 3 of Ref. 2 with Figs. 33 and 36 of Ref. 5, indicates that the total position error with VARPAR is at least 10 times less than the in-track error with the best of the schemes described in Ref. 5. Of course, the accuracy of any of these schemes could be improved by the updating procedure that we advocate here.

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Technical Comments

Comments on "Some Energy and Momentum Considerations in the Perforation of Plates"

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The equations governing the macroscopic behavior of a plate-projectile impact situation, commonly called plugging, were developed by Giere.[1] The two-body, dead or plastic impact equations were shown to result from an analysis of the perforation of thin plates by projectiles impacting at ordnance velocities if the kinetic energy and momentum transferred to the target are considered to be negligible. This analysis assumes that the projectile and plug leave the plate with a common velocity. The residual velocity of the projectile, compared with the initial velocity, was given as

$$v_r/v_s = (1 + \alpha)^{-1} \tag{1}$$

where α is the ratio of the mass of the plug to the mass of the projectile. Plugging has often been advocated as a model for systems wherein the materials are ductile, the projectile is blunt, the target thickness is small compared with the length of the projectile, and the projectile is initially traveling at ordnance velocities greater than the minimum perforation velocity. Although some of the restrictions on this theory were pointed out by Giere, the limits that they impose have not been demonstrated.

There are data available which can be compared with the theory of Eq. (1) if it is assumed that the projectile and plug have the same cross-sectional area.[2–5] The data used in Fig. 1 for each point were the average of at least four tests with impact velocities between 50% greater than the minimum perforation velocity and 1 km/sec, except for that of Ref. 3. Spells' data were all taken at 1.469 km/sec, and it demonstrates the trend of closer agreement with the theory at higher velocities which the other data showed.

It is apparent that these data are incompatible with the assumption of negligible energy loss from the projectile-plug system as used in the development of Eq. (1). Giere's analysis suggests that a possible explanation for this disagreement

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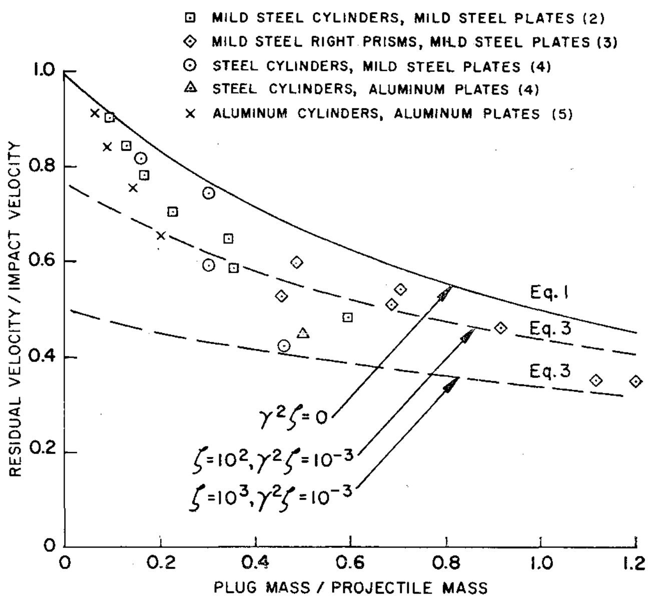

Fig. 1 Projectile velocity ratio for plate perforations.

is the nonkinetic energy going into the target E_0. However, if the plate momentum and energy are equal to zero, then $E_0 = 0$ if the energy and momentum of the system are to be conserved. If the momentum acquired by the target is not excluded from the system, the nonkinetic energy W can be expressed as

$$W = E + E_0 = \frac{mv_s^2}{2}\left[1 - \frac{1 + \alpha + \gamma^2\zeta}{(1 + \alpha + \gamma\zeta)^2}\right] \tag{2}$$

where, in addition to Giere's notation, $\gamma = V/v_r$ and $\zeta = (M - m_t)/m$. The inclusion of the plate in the system results in the following equation for the velocity ratio of the projectile:

$$v_r/v_s = (1 + \alpha + \gamma\zeta)^{-1} \tag{3}$$

Although it is realistic to assume that, in comparison with the projectile, the target acquires negligible energy, because the relative mass of the target is so great, it does not necessarily follow that its momentum is negligible. The effect of the target momentum is shown in Fig. 1 by the dashed lines that represent targets that have acquired 0.1% of the residual kinetic energy of the projectile and have masses 100 and 1000 times as large as that of the projectile. These systems have a ratio of momentum to kinetic energy of 316:1 and 1000:1, respectively. In systems with large differences in the relative mass of the components, one should be cautious before assuming that the momentum and kinetic energy of any one component is negligible.

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Comment on "Angle of Attack from Body-Fixed Rate Gyros"

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The condition $\dot{\alpha} = 0$ when $\dot{\Omega} = 0$, which Nelson[1] used to develop a technique for measuring the angle of attack of a symmetric, spinning body, can be alternately derived without algebraic manipulation. A clearer physical interpretation of the vehicle motion is obtained by the alternate method.

Instead of the Euler equations, use the equation of motion derived by Nidey and Seames[2] for a coordinate system rotating with the transverse angular velocity:

$$M_\tau = I_\tau \dot{\omega}_\tau \tag{1}$$

$$M_\nu = I_\tau \omega_\tau \Omega_\lambda - I_\lambda \omega_\lambda \omega_\tau \tag{2}$$

$$M_\lambda = I_\lambda \dot{\omega}_\lambda \tag{3}$$

where $\dot{\omega}_\lambda \equiv \dot{\Omega}$. We obtain the result $\dot{\Omega} = 0$ when $\epsilon = \phi$ (or $\epsilon = \phi + 180°$) by inspection of Eq. (1) since the latter equality implies $M_\tau = 0$. Thus, the need for algebraic manipulation is eliminated.

A symmetric body at a preatmospheric altitude precesses with the well-known free-body motion. After entering the atmosphere the body is constrained by the influence of the aerodynamic moment to precess about an axis other than that formed by the moment of momentum vector. In fact, the body precesses with the angular velocity $\boldsymbol{\omega}_p$ (or $\boldsymbol{\Omega}$ in notation of Ref. 2) the same as the τ, ν, λ coordinate system. Because the coordinates and the body share the transverse component of angular velocity

$$\boldsymbol{\omega}_p = \Omega_\lambda \boldsymbol{\lambda} + \omega_\tau \boldsymbol{\tau} \tag{4}$$

where Ω_λ can be found from Eq. (2).

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Comment on "Motion of the Center of Gravity of a Variable-Mass Body"

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In a recent note,[1] Punga has shown the effects of a moving center of mass on the equation of motion of a variable mass body. The result that Punga was seeking can be found in the literature,[2–5] but his result disagrees with previously published results. It is this author's contention that the premise upon which Punga's paper was based is false.

Punga's basic premise was that the equation of motion for a variable-mass system can be expressed in the following form:

$$\int (d^2\mathbf{R}/dt^2)dm = \mathbf{F} + \mathbf{K} \tag{1}$$

where the integration is extended over the mass of the body at time t. He defines $\mathbf{F}$ as the external force acting on the body and $\mathbf{K}$, the reactive force acting on the body which is produced by mass ejection. Equation (1) is an extension of Newton's second law to any body, but should not include the term $\mathbf{K}$ since this imaginary force is a by-product of the left-hand side of the equation.

Thorpe[2] has derived a relation for the motion of the center of mass of a variable-mass body in which he started with the classical formulation of Newton's second law, namely,

$$\mathbf{F} = \int \mathbf{a}\,dm \tag{2}$$

where $\mathbf{a}$ is the acceleration of the element of mass dm. Thorpe's result is as follows:

$$\mathbf{F} = \frac{d}{dt}(M\mathbf{V}^*) + \int_s \rho\mathbf{u}(\mathbf{u} - \mathbf{v})\cdot\mathbf{n}\,ds + \frac{d}{dt}\int_s \rho(\mathbf{r} - \mathbf{R}^*)(\mathbf{u} - \mathbf{v})\cdot\mathbf{n}\,ds \tag{3}$$

where $\mathbf{V}^*$ is the velocity of the mass center, $\mathbf{u}$ is the absolute velocity of the mass particle, and $\mathbf{v}$ is the velocity of the boundary. Let us denote the absolute velocity of the escaping gases as $\mathbf{v}_e$, and from physical reasoning the mass rate of flow across the boundary can be written as

$$\frac{dM}{dt} = -\int_s \rho(\mathbf{u} - \mathbf{v})\cdot\mathbf{n}\,ds \tag{4}$$

Let us define a new quantity

$$\mathbf{R}_N \frac{dM}{dt} = \int_s \rho\mathbf{r}(\mathbf{u} - \mathbf{v})\cdot\mathbf{n}\,ds \tag{5}$$

Then with these new definitions, Eq. (3) can be reduced to

$$\mathbf{F} = M\frac{d\mathbf{V}^*}{dt} + \frac{d^2M}{dt^2}(\mathbf{R}^* - \mathbf{R}_N) + \frac{dM}{dt}\left(2\frac{d\mathbf{R}^*}{dt} - \frac{d\mathbf{R}_N}{dt} - \mathbf{v}_e\right) \tag{6}$$

Equation (6) has been derived by Rankin[3] and Leitmann[4] by using an alternate form of Newton's second law.

Equation (6) can be reduced further by using an intermediate frame of reference fixed in the body at point 0, in a manner

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